Hydrodynamic Stabilization of Supercavitating Underwater Bodies

Oleksandr Mayboroda*

Kyiv State Maritime Academy, Ukraine

Abstract

In order to stabilize the motion of supercavitating underwater bodies, the damping action of the stern elastic interceptors which glide along the cavity surface is used. The results of calculation of normal forces on planing interceptors at the Mach numbers below 1.5 with a detached and attached shock wave are obtained. These results agree satisfactorily with known theoretical solutions and experimental data.

Keywords: supercavitating bodies; stabilization of moving; gliding at the Mach numbers below 1.5

Introduction

The increase the speeds of ships and other technical objects moving in the water calls new problems for hydrodynamics, in particular, the need to take into account the water compressibility, including at supersonic speeds [1]. The maximum achieved body velocity in water is about 1550 m/s and according to known data the speed range up to 2000...2200 m/s, that is corresponding to the Mach numbers below 1.5 (the Mach number is the ratio of the body speed to the sound speed in the fluid), is considered to be promising for creation of high-speed underwater bodies. Under these conditions, compressibility of water is accompanied by known gas-dynamic effects – compression shocks, shock waves etc.

The technical disadvantage of supercavitating bodies is the movement instability, since the application of resistance force in the bow stagnation point precludes the hydrodynamic damping. To stabilize the movement of such bodies, it is possible to install the stern elastic plates – planing interceptors that glide on the cavitation cavity surface and create a hydrodynamic damping. Gliding on the surface of incompressible water, that is, at the Mach numbers below 0.3, is sufficiently studied. High-speed gliding into account the gas-dynamic properties of water has not been considered yet for lack of applied interest.

Hence, it is necessary to investigate the hydrodynamic characteristics of planing plates with finite aspect ratio those glide in the range of sub-, trans- and supersonic velocities.

Body

The compressibility of water as a condensed medium, unlike a gas, has some characteristic features. The equilibrium distances of water molecules correspond to a minimum of the potential energy of their interaction. When such a medium is compressed, the internal pressure fast increases and, unlike gases, has a nonthermal, only elastic nature. This determines the main features of the water behavior during compression.

At pressures up to $3{\cdot}10^9$ Pa, corresponding to the Mach numbers below 1.5, the water state equation has a form Teta [2] with the isentropic index γ =7.15. Consequently, under the considered conditions, water is a barotropic medium, and its internal energy is the sum of two functions, one of which depends only on the specific volume, and the other – only on the entropy. Therefore, it can be shown that isentropic processes in water are isothermal. Comparison of the isentropic flow relationships of pressure and density for water and air shows that with increasing the Mach number the water exhibits under compression more greater elasticity than air.

1. Subsonic flow

According to Wagner's analogy [3], a flow under the planing plate corresponds to that under a lower surface of a thin wing. For such a surface, experimental data on the load in a wide range of the Mach numbers are known [4] and by the method of local linearization the Mach number effect on the load increase was estimated [5].

As it is shown by Kusakava [6], a pressure distribution on the lower surface of a thin wing at the Mach numbers below 1 is affinity-like. So it can be assumed that the pressure distribution on the planing plate in the subsonic flow

*Corresponding Author, Oleksandr Mayboroda: olexander.mayboroda@gmail.com

will also be affinity-like with the isentropic similarity factor k_M, which can be determined from the isentropic flow relationship as

$$k_M = \frac{2}{\gamma M^2}\left[\left(1+\frac{\gamma-1}{2}M^2\right)^{\frac{\gamma}{\gamma-1}}-1\right]. \tag{1}$$

In Fig. 1 the Mach number effect on the relative change in the normal force coefficient on the profile lower surface according to the proposed similarity factor k_M for water and air in comparison with the well-known Prandtl-Glauert rule [2], experimental data [4] and the results of a theoretical estimate of such effect [5] is showed.

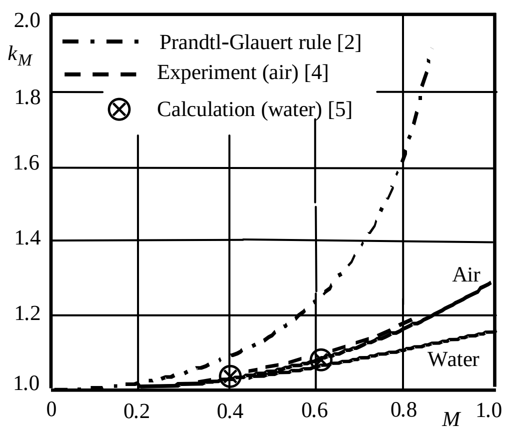

Fig. 1 Effect of the Mach number on hydrodynamic load of a profile

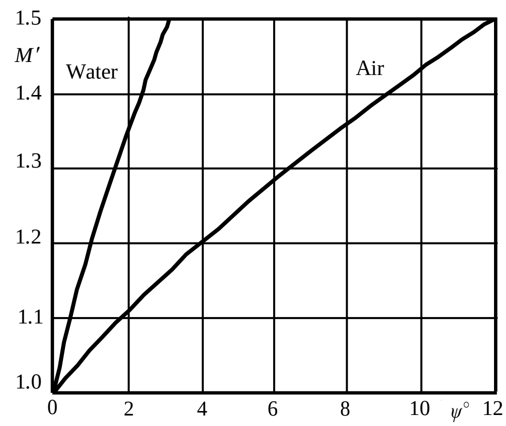

Fig. 2 Effect of a running pitch angle ψ on the Mach number M' of the shock wave attachment

Satisfactory agreement of these results allows us, for the indicated flow conditions, to recommend the similarity factor k_M (1) for taking into account the compressibility of various media at the Mach numbers below 1. In contrast to the Prandtl-Glauert rule, the similarity factor k_M takes into account the media thermodynamic properties and, as it is shown below, can be generalized to supersonic regimes with a detached shock wave.

Lack of finite-span nonlinear effects during gliding allows to generalize the well-known Jung theory [4] and to obtain the expression for the derivative of normal force C_n^ψ for a planing plate

$$C_n^\psi = \frac{\pi k_\psi k_M}{1+2\lambda k_\psi k_M}, \quad M \le 1,\ \psi \le 60^\circ,\ \lambda \ge 0.01, \tag{2}$$

where λ is an aspect ratio of a plate; k_ψ is a coefficient of effect of the running pitch angle ψ according to the nonlinear theory of Sedov [7].

Generalizing the well-known Wagner model [3] for the case of subsonic gliding, taking into account the proposed coefficient k_M (1), one can obtain an expression for a value of the relative bow backwater flow

$$\frac{l}{l_0} = \frac{1}{2}k_M\left(3+\sqrt{1+\frac{1}{\lambda_0}}\right)-1, \quad M \le 1,\ \psi \le 60^\circ,\ \lambda \ge 0.01, \tag{3}$$

where l is a wetted plate length; l_0 and λ_0 are a length and initial aspect ratio of the plate relative to an undisturbed fluid surface.

The affinity-like distribution of the pressure with increasing of the Mach number suggests that the relative position of a pressure centre on the plate remains.

2. Supersonic flow

When a plate moves with supersonic speed, a shock wave moves ahead of it. The isentropic character of the water state equation makes it possible to consider a formation and propagation of this shock wave in the quasi-acoustic approximation and use the well-known dynamic compatibility conditions [8]. Such a problem has a self-similar solution only for a sufficiently weak shock [8]. Since there is no further increase of the pressure on the plate behind

*Corresponding Author, Oleksandr Mayboroda: olexander.mayboroda@gmail.com

the shock, we can assume that it belongs to the weak type. Taking into account the lack of a mixed flow under the plate we admit the existence of a quasi-stationary solution of the problem. An additional difficulty is a presence of a free fluid surface. At the intersection point of the shock front with the free surface, the centered Prandtl-Meyer expansion wave emerges, and the flow turns in the direction of the free surface [9].

In view of the problem complexity, first we analyze the simplified case – the gliding of the plate with infinite aspect ratio, so called the profile.

When the plate is gliding with a supersonic speed it is possible three characteristic regimes of flow around it:

$1 < M < M'$ – there is a subsonic flow behind the detached shock wave and at M' the shock wave attaches to the plate;

$M' \le M \le M''$ – there is a subsonic flow behind the attached shock wave, where M'' is the Mach number at which the velocity behind the shock wave becomes supersonic;

$M'' < M$ – there is a supersonic flow behind the oblique shock wave.

As it is shown in the works on supersonic gas jets [10], the limiting angles of the flow deflection in an attached shock wave are also preserved for flows with a free surface. Under these assumptions, an approximate calculation of these angles in water and air was made and is shown in Fig. 2. As it can be seen, the planing profile at running pitch angles $\psi \ge 2.6^\circ$ for the Mach numbers below 1.5 is always streamlined with an attached shock wave.

For water, within the applicability of the Teta state equation, the similarity laws of the perfect gas dynamics can be applied when the adiabatic exponent of gas is replaced by a value γ in the similarity criteria. That allows using for water the results of exact solutions and experiments obtained for similar gas flows.

In particular, an approximate evaluation of the shock wave attachment in water, shown in Fig. 2, by the value of the Karman-Cheney similarity criterion [8] agrees well with the solution of a similar problem obtained by Vincenti and Wagoner [11] by the hodograph method.

2.1 Planing profile with detached shock wave

At $1 < M < M'$ the shock wave *C* (Fig. 3) is disconnected and interacts with the fluid free surface *A*. Such a flow exists always at the running pitch angles $\psi \ge 2.6^\circ$ of the profile *L*. The shock is curved and in the convex head part, turned towards the profile movement, is straight. Supersonic flow in the main part of the shock turns into subsonic. In the remaining part, the shock intensity decreases, the flow behind it remains supersonic and the shock decomposes into a weak discontinuity. From a free surface, the shock is reflected by a centered Prandtl-Meyer expansion wave *PM* and the flow in surface vicinity remains supersonic. At the point of reflection, the pressure decreases to an unperturbed value, and the free surface deformation is determined by the distance from the shock to the profile, the so-called thickness of the shock layer.

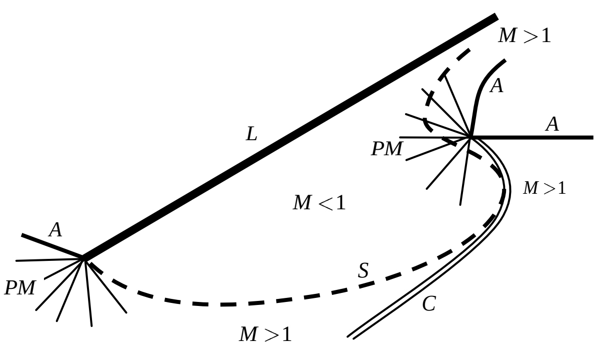

Fig. 3 The planing profile with the detached shock wave

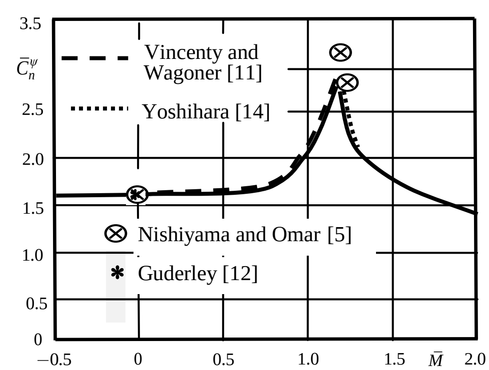

Fig. 4 Hydrodynamic load on the profile at transonic gliding with the running pitch angles below 2.6°

The profile is streamlined by the subsonic flow behind a direct shock up to the after-body Prandtl-Meyer expansion wave. From supersonic flow below and in the backwater flow on the free surface, the subsonic flow is detached by the contact discontinuity line *S*.

*Corresponding Author, Oleksandr Mayboroda: olexander.mayboroda@gmail.com

Using the correspondence principle [12] for the flow behind a straight shock wave we can construct in this case the following generalization of similarity factor k_M :

$$k_M = \frac{2}{nM^2}\left[\frac{\left(\frac{n+1}{2}M^2\right)^{\frac{n}{n-1}}}{\left(\frac{2nM^2}{n+1}-\frac{n-1}{n+1}\right)^{\frac{1}{n-1}}}-1\right], \quad 1<M<M', \quad \psi \geq 2{,}6^\circ. \tag{4}$$

For the supposed assumptions, it can be suggested that in the range of the Mach numbers $1<M<M'$ the change in the relative position of the pressure centre of the planing profile will be insignificant. According to Vincenti and Wagoner [11], for example, the displacement of the pressure centre to the trailing edge in this range is 4% of the profile chord.

2.2 Planing profile with attached shock wave

Such a flow exists always at the running pitch angles of the profile $\psi \leq 2.6^\circ$. As it is known, there is no stagnation point on the profile [13]. The flow behind the shock at $M = M'$ is subsonic and with a velocity increase at $M = M''$ becomes supersonic. The value of the interval $\Delta M = M'' - M'$ depends on the Mach number and for $M \leq 1.5$ in water does not exceed 0.05. By the smallness of this interval we can take $\Delta M = 0$ and as the Mach number M_S of the shock attachment consider the value $M_S = M'$.
In this case, the flow around the planing profile is similar to that of a thin symmetrical wedge and, in order to calculate the derivative of normal force C_n^ψ , we can use the corresponding results.

In Fig. 4 in the transonic similarity criteria $\bar{C}_n^\psi$ and $\bar{M}$ [8] a comparison of profile loads calculation in the range of $0.95 \leq M \leq 1.25$ according to the proposed method with known results of Nishiyama and Omar [5], Vincenti and Wagoner [11], Guderley [12] and Yoshihara [14] is showed. Here the value $\bar{M} = 0$ corresponds to $M = 1$.
At $M \geq M''$ the pressure centre of the planing profile is displaced to the middle of the wetted length. In the range $M' \leq M \leq M''$ the relative location of the pressure centre can be improved by interpolation on the basis of known statement on the continuity of the shock attachment with increasing flow velocity [12].

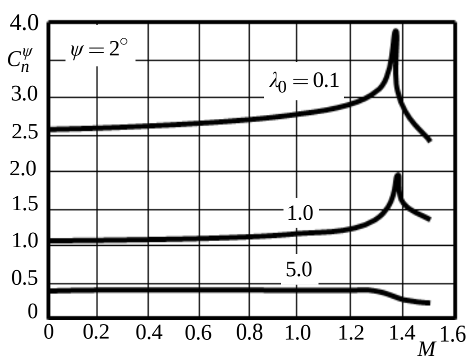

Fig. 5 Hydrodynamic load on plates at gliding with the attaching of the shock wave

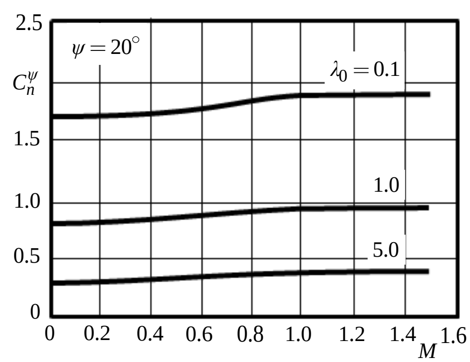

Fig. 6 Hydrodynamic load on plates at gliding with the detached shock wave

2.3 Planing plate with finite aspect ratio

The subsonic flow behind the detached shock wave in the quasi-acoustic approximation is isentropic and, therefore, potential. If to use the similarity factor k_M (4) one can apply in this case the expression (3) for the value of the relative bow backwater flow on the planing plate. Behind an attached shock wave there is no an effect of the bow backwater flow on a planing plate because the stagnation point is missing here.

*Corresponding Author, Oleksandr Mayboroda: olexander.mayboroda@gmail.com

Accounting of the aspect ratio influence on the hydrodynamic load of the planing plate at supersonic movement with the detached shock wave can be carried out with the generalized Young formula (2) and with an attached shock wave by the Hilton method [4].
Hydrodynamic loads on planing plates with different initial aspect ratio and running pitch angles at the Mach numbers below 1.5 are showed in Fig. 5 and 6. The values of the quantity C_n^{ψ} at $M = 0$ in Fig. 5 and 6 correspond to the known results for the gliding over the incompressible fluid surface.

Conclusion

For an isentropic equation of state of water, a quantitative estimate of the compressibility effect on the gliding of the plate at the Mach numbers below 1.5 with a detached and attached shock wave is first obtained.
A rule for conversion the pressure coefficients for the incompressible fluid flowing under the profile lower surface to their values for a given Mach number below 1.5 is proposed. Unlike the well-known Prandtl-Glauert rule, the proposed approach takes into account the thermodynamic properties of the medium.
Hydrodynamic loads on planing plates with initial aspect ratio λ more 0.01 and running pitch angles ψ below 60° at the Mach numbers below 1.5 with detached and attached shock wave are gotten.
The obtained results are in satisfactory agreement with known theoretical solutions and experimental data and can be used to calculate the damping forces on interceptors of supercavitating underwater bodies.

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*Corresponding Author, Oleksandr Mayboroda: olexander.mayboroda@gmail.com