Cavitation in Sudden Gap Expansion as a Model for Synovial Joint Cavitation

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Abstract

The sound of cracking joints, more commonly knuckles, is thought to be due to the cavitation and subsequent bubble collapse in the synovial fluid of the joint. This fluid is shear-thinning and rheopectic, confined within a synovial membrane sac, and surrounded by complex curved surfaces comprised of cartilage on bone. A simplified model potentially capturing the dominant physics of this problem is cavitation between axisymmetric parallel disks in an infinite domain. Two analytical models are found for this problem: the Stefan-Reynolds equation and Kuzma’s derivation. Analytical solutions for flow and pressure distributions are found using these simplified models. The disks are suddenly accelerated from an initial separation of a sub-mm gap, and the pressure at the center decreases to values well below the Blake-threshold, leading to cavitation. The initial work reported in this paper utilized Newtonian fluid, namely water, and is focused on comparing the observed and expected cavitation dynamics.

Keywords: cavitation; biofluid

Introduction

It is widely believed that fluid mechanics, namely cavitation, is responsible for popping or cracking sound of synovial joints (Unsworth et al. 1971; Kawchuk et al. 2015; Dowson et al. 1970; Watson et al. 1989). This cracking sound occurs when bones are pulled apart axially, resulting in formations of gas or vapor bubble within the joint. Other joint movement can also produce sounds, but may be due to tendons or ligaments moving across bone. Cracking is studied in metacarpal joints, commonly known as knuckles (Unsworth et al. 1971; Kawchuk et al. 2015), but the same mechanism also occurs in knee, hip, and spine joints. It is also assumed that understanding more about joint geometries and synovial fluid composition may lead to early detection of joint disease, such as arthritis (Unsworth et al. 1971; Cooke et al. 1978; Fam et al. 1978). Synovial fluid reduces the friction between the cartilage covering the bone, as surfaces move across each other, and transports nutrients, waste (Balazs et al. 1967; Cooke et al. 1978). In a healthy subject, synovial fluid is opaque in color, similar to blood serum, and shear-thinning (Balazs et al. 1967; Fam et al. 2007). Furthermore, long polymer chains (hyaluronan) in the fluid and attached to the cartilage give the fluid rheopectic properties (Cooke et al. 1978; Rwei et al. 2007).

Unsworth et al. (1971) setup an apparatus to pull the index finger of human subjects and recorded the bone separation of the joint using x-ray exposures. They determined that cavitation collapse is responsible for the cracking noise, and observed that about 15 min after the crack the bone separation distance returned to normal. This is presumed to be following Henry's Law, as the gases redissolved into the fluid (Kimbrough 1999). Gas content was on average 15 percent at room conditions, with 80 percent of the gas being CO2. In Unsworth et al. (1971) a joint simulator was used to study the radius of curvature of the two joint surfaces, and it was found that the synovial fluid composition and joint geometry are the two main factors determining whether a joint will crack. Kawchuk et al. (2015), similarly to Unsworth et al. (1971), pulled the index finger of subjects, but used Magnetic Resonance Imaging (MRI) to image the joint. At a frame rate of ~3Hz, they were able to record a bubble in the synovial fluid at the time of crack. This bubble was reported to disappear in unspecified time once the axial force was no longer applied. The cracking sound was attributed to the bubble inception, or trionucleation, rather than its collapse (Kawchuk et al. 2015).

The apparent lack of full agreement and thorough fundamental fluid mechanical explanation motivated the present study. Beginning with a canonical geometry to analyze and experiment with, we seek to investigate the mechanisms that produce the sound in more detail, and attempt to glean useful physical insight about joint cracking grounded solidly in fluid mechanics.

Theoretical and Experimental Model

The real joint is quite complex, with the cartilage surface being porous and having non-symmetric shape, and the synovial fluid is non-Newtonian confined within a membrane. So, we begin our investigation on a simplified model of this flow. While curved geometries and non-Newtonian fluids will eventually be utilized, the first geometry we study is the canonical form of two flat disks in an infinite fluid domain, shown in Figure 1(a).
Experimental data is obtained, and we utilize an analytical model to predict the pressure distribution, and more importantly, the minimum pressure achieved at the center of the discs. We calculate the minimum pressure at the center of the discs based on boundary conditions and the prevailing flow conditions extracted from experimental footage.

**Theory**

The available mathematical models, as summarized by Engmann et al. (2005), are analyzed, and the two most suitable models are discussed here. The first mathematical model is based on a squeeze flow derivation by Stefan and Reynolds. This formulation has been adapted for non-Newtonian fluids with a power-law shear-stress expression by Scott (1931), and re-derived as an explicit pressure formulation by Leider (1974) (Equation 4). While this derivation is applicable to non-Newtonian fluids, it fails to properly account for inertial effects. To account for the effects of fluid inertia in separating plates, Jackson (1962) derived solution with corrections from Kuzma (1967), which results in a pressure distribution given by Equation 9.

The formulation by Leider is based on an integral mass balance of the fluid contained between the disc faces and the radial component of the Cauchy momentum equations. One assumes incompressible flow and quasi-steady-state flow conditions, allowing the local and instantaneous description of the flow to be steady-state between two parallel plates with negligible inertial terms and normal stresses. $R \gg h$ and $u_r$ is the dominant velocity component. This derivation starts from the integral mass balance of the fluid between the two rod faces

\[ \int_0^h u_r \, dz = -\frac{r h}{2}. \]  

And, the radial component of the Cauchy momentum equations

\[ \frac{\partial u_r}{\partial t} + u_r \frac{\partial u_r}{\partial r} + u_\theta \frac{\partial u_r}{\partial \theta} + u_z \frac{\partial u_r}{\partial z} - \frac{u_\theta^2}{r} + \frac{1}{\rho} \frac{\partial \rho}{\partial r} + \frac{1}{\rho r} \frac{\partial (r \tau_{rr})}{\partial r} + \frac{1}{\rho r} \frac{\partial \tau_{\theta r}}{\partial \theta} + \frac{1}{\rho r^2} \frac{\partial \tau_{rr}}{\partial \theta} - \frac{\tau_{rr}}{r^2} + g_r. \]  

With $\dot{h}$ representing the velocity of the upper rod surface, neglecting body forces $g_r$, all inertial terms, the normal stresses involving $\tau_{rr}$ and $\tau_{\theta \theta}$, and the shear stress $\tau_{\theta r}$ due to axial symmetry and using a rheological formulation for shear stress $\tau_{rr}$ for power-law fluids is given by

\[ \tau_{rr} = -m \left( \frac{\partial u_r}{\partial z} \right)^{n-1} \frac{\partial u_r}{\partial z} = m \left( \frac{\partial u_r}{\partial z} \right)^n. \]  

Where $m$ is the flow consistency index and $n$ the flow behavior index (Leider 1974). With Equations 1-3, we obtain the explicit formulation for the pressure difference according to Leider (1974). The pressure difference for a power-law fluid according to Leider can then be derived through substitution of the shear stress into Equation 2 and

![Figure 1](image1.png)  

Figure 1. a) Coordinate system for pressure calculations. At time $t=0$, top rod is moved up while the bottom is stationary, and is modeled as symmetric about the z-axis, b) Experimental setup with force actuation mechanism, test section, light and high-speed video camera, c) Sample image from high-speed video capturing a large cavitation event during rapid gap expansion, image width $w = 0.7 d_r$. 

after rearranging and integrating the result can subsequently be substituted into Equation 1, which when integrated yields

\[
p - p_{atm} = \frac{(-\dot{h})^n}{h^{2n+1}} \left( \frac{2n + 1}{2n} \right)^n \frac{m R^{n+1}}{n + 1} \left[ 1 - \left( \frac{r}{R} \right)^{n+1} \right].
\]  

(4)

Kuzma’s formulation is based on the continuity equation in differential form and the momentum equation in the radial direction. He assumed incompressible flow, \( R >> h \), and \( u_r \) as the dominant velocity component with negligible azimuthal and vertical pressure gradients resulting in the pressure depending only on \( r \) and \( t \). The starting point is the differential form of the continuity equation

\[
\frac{1}{r} \frac{\partial (ru_r)}{\partial r} + \frac{\partial u_z}{\partial z} = 0,
\]

(5)

and momentum equation for Newtonian fluid flow in this geometry

\[
\rho \frac{\partial u_r}{\partial t} + \rho u_r \frac{\partial u_r}{\partial r} + \rho u_z \frac{\partial u_r}{\partial z} = -\frac{\partial p}{\partial r} + \mu \left( \frac{\partial^2 u_r}{\partial r^2} + \frac{1}{r} \frac{\partial u_r}{\partial r} - \frac{u_r}{r^2} + \frac{\partial^2 u_r}{\partial r^2} \right).
\]

(6)

Obtaining the initial guesses for the velocity components from the lubrication assumption

\[
u_r(0) = \frac{3r\dot{h}}{h^3}(z^2 - hz), \quad u_z(0) = -\frac{\dot{h}}{h^3}(2z^3 - 3hz^2)
\]

(7)

and substituting into Equation 6, this can be rearranged and integrated yielding an expression for \( u_r \). Subsequently, this can be substituted into the integral continuity equation

\[
\int_0^r u_r \, dz = -\frac{r\dot{h}}{2}
\]

(8)

Equation 8 can then be rearranged to yield an explicit formulation for the pressure gradient. Assuming a constant ambient pressure and no-slip conditions at the surfaces, integration yields the pressure difference for Newtonian flow according to Kuzma as

\[
p - p_{atm} = -\frac{r^2 - R^2}{2} \left( \frac{6\dot{h}}{h^3} + \frac{3\dot{h}}{5h} - \frac{15\dot{h}^2}{14h^2} \right).
\]

(9)

It has been noted that inertial effects become significant for high \( Re = \rho h^2/\mu \), with \( h \) being the surface separation, \( \dot{h} \) the upper surface velocity, \( \rho \) the density and \( \mu \) the viscosity (Jackson 1962). While the lubrication assumption inherent in the solution is only satisfactory for small Reynolds numbers, Kuzma’s experiments show close agreement up to Reynolds numbers of \( Re = 60 \) (Kuzma 1967). That is, while the model proposed by Kuzma relies on assumptions for the velocity profile, which are no longer wholly satisfactory for higher Reynolds numbers, it shows unexpectedly good agreement with experimental results (Kuzma 1967). While Leider’s model is also applicable to non-Newtonian flows, it assumes steady-state conditions and neglects inertial effects, thus making it less suitable for the problem of rapidly separating discs with high accelerations and high Reynolds number flows. Hence, while the present paper only discusses the previous models, we are deriving a model suitable to both non-Newtonian flows and high Reynolds number flows with large accelerations.

**Experimental Setup**

The experimental setup mirrors the theoretical model, with two axisymmetric discs that rapidly separate. The working fluid for reported experiments was water nominally at 20°C and, for these initial experiments, presumed saturated at 1 atm. A clear 76mm diameter acrylic tube is sandwiched between two aluminum plates, with the seal made watertight using a rubber gasket. In the tube are two 16mm diameter stainless steel shafts with polished ends. One shaft is fixed to the bottom plate, and the other is free to move axially. A repeatable force is transferred through a lever mechanism from free-falling weights dropped from a chosen height to the top shaft by an 80/20 structure. The setup is backlit with 200 Watt LED light, and images are recorded at 110,000fps with a Vision Research Phantom v1210 high-speed camera (Figure 1b).
Results

High speed videos were recorded at 110,000fps for each experiment, and sample time series are shown in Figure 2, which shows a zoomed out view (a) and a higher resolution image sequence (b) of a typical experiment. We observe bubble formation and slight growth at the instant of calculated minimum pressure. The bubbles continue to grow more rapidly until reaching a maximum radius followed by rapid collapse, as expected. During collapse and rebound at the center, we can see secondary bubbles forming at other points in the domain in 2(a), presumably due to the shock-wave created by the initial bubble collapse.

![Initial Gap - Minimum Pressure - Begin Bubble Expansion]

| t = 0 ms | t = 0.8 ms | t = 0.8625 ms |

| Bubble Growth - Maximum Bubble Radius - Bubble Collapse |

| t = 0.9875 ms | t = 1.4375 ms | t = 1.525 ms |

![Initial Gap - Minimum Pressure - Begin Bubble Expansion]

| t = 0 ms | t = 0.87273 ms | t = 0.97273 ms |

| Bubble Growth - Maximum Bubble Radius - Bubble Collapse |

| t = 1.0636 ms | t = 1.1545 ms | t = 1.2 ms |

Figure 2: a) Time sequence of an experiment, image width \( w = d_{rod} \), b) Time sequence of zoomed in experiment, image width \( w = 0.3d_{rod} \)

The experiment shown in the time-series displayed in Figure 2(b) is analysed using image tracking algorithms. We analyse the position, velocity and acceleration of the upper rod surface, with results displayed in Figure 3. The Reynolds number, relative pressures and cavitation numbers, based on both Kuzma’s as well as Leider’s analytical models, are presented in Figure 4. When comparing Figure 2(b) to the displacement in Figure 3(a) we can see that the minimum pressure occurs just after the initial movement of the rod. At this moment in time the gap height is still close to the initial value. While the velocity is still low, the acceleration has reached \( a \approx 600 \, \text{m} / \text{s}^2 \). This highlights the relative importance of the gap height and inertial terms, categorized by the acceleration versus the rod velocity.

In Figure 4, which compares the relative pressures based on Kuzma’s and Leider’s analytical models, the cavitation number, \( \sigma \), is defined as

\[
\sigma = \frac{p_{center} - p_c}{\frac{1}{2} \rho h_{max}^2}
\]

(10)

Where \( p_c \) is the Blake-threshold based on an initial bubble radius of 1 \( \mu \text{m} \) and the surface tension and vapor pressure of water at 1 atm and 21°C (Brenner 2005). With \( p_{center} \) as the pressure in the radial and vertical center of the fluid domain and \( h_{max} \) as the maximum velocity of the upper rod during the experiment.
Figure 3: a) Position of the upper and lower rod, measurement points and spline fit, b) Velocity of upper rod, based on spline fit, c) Acceleration of upper rod based on spline fit. The vertical dash-dot line indicates moment cavitation was first observed.

Figure 4: a) Fitted position of upper rod, b) Reynolds number, c) Relative pressure and Blake-Threshold Kuzma model, d) Relative Pressure and Blake-Threshold Leider model, e) Cavitation number Kuzma model, f) Cavitation number Leider model. The vertical dash-dot line indicates moment cavitation was first observed.
Comparing both Leider and Kuzma models highlights the disadvantages of neglecting inertial terms for a case such as ours. While Equation 9 predicts a cavitation event to occur slightly before cavitation is evident in the high speed video, the pressure change Equation 4 is orders of magnitude lower and does not predict cavitation. We should also note that Equation 4 assumes steady state flow conditions, and while Figure 4(d) indicates that this is not a valid assumption for our experiment, Engman et al. (2005) have discussed this model’s ability to produce satisfactory results for compression cases with low Reynolds numbers. However, once cavitation occurs, the underlying assumptions regarding incompressibility and constant density are no longer valid. Therefore, the equations can no longer expected to correctly describe the flow field in the gap after the occurrence of initial cavitation (indicated by blue vertical line and dashed relative pressure and cavitation number lines in Figures 3 and 4). The dynamic pressure transducer used for next stage of the experiments will shed light on the pressures experienced during the cavitation event and the rebound of the cavitation bubbles.

Conclusion

A simplified experiment was designed to study of cavitation in joints. Adapted and rederived analytical solutions predicted the pressure minima with varying success, and results show that inertial terms cannot be neglected for the given canonical geometry and large accelerations. The experimental setup also allowed for repeatable measurements and yielded data consistent with prediction of the beginning of the cavitation event using Kuzma’s model.

In the next stage the authors will incorporate direct dynamic pressure measurements, vary dissolved gas contents, and transition to study shear thinning fluids with adapted mathematical modeling. The effect of confinement due to a flexible membrane that mimics the synovial sac and curved interfaces reminiscent of the ball-and-socket geometry of a real joint will also be studied, if resources enable this. As it has been speculated (Unsworth 1971) that cracking joints can cause arthritis via mechanisms similar to that damaging ship propellers as vapor bubbles collapse near the surface, an investigation into joint cavitation from a fluid mechanics perspective may give us insight into how cracking affects joints. It has also been proposed that the cracking sound could be used as an initial diagnostic tool to reveal information about an individual’s synovial fluid composition and joint topology (Cooke et al. 1978). While evaluating the validity of these is beyond the present and planned scope of study, the fundamental understanding from these laboratory experiments and theory may offer guidance for medical applications.

References